ECIS Mathematics League 2005

Fourier League (suggested age 16-18)   No Calculators

Section I – 40 minutes

1. The length of each edge of a cube is increased by 25%. Find the corresponding percent increase of the surface area of the cube.

2. There is a riddle about Diophantus, a 4^{th} century Greek mathematician, concerning how long he lived. Determine how long Diophantus lived from the riddle:

Diophantus's youth lasts 1/6 of his life. He grew a beard after 1/12 more of his life. After 1/7 more of his life, Diophantus married. Five years later, he had a son. The son lived exactly half as long as his father, and Diophantus died just four years after his son's death. All of this totals the years Diophantus lived.[1]

3. A football (soccer ball) is a convex polyhedron composed of 12 pentagons and 20 hexagons. A vertex is a point of intersection of three polygons on the ball, and an edge is a line segment connecting two vertices. Determine the number of edges on a football.

4. If i is defined to be the imaginary number such that $i^2 \equiv -1$, find the sum of the following sequence: $i + i^2 - i^3 + i^4 - i^5 + i^6 - \cdots + i^{100}$

5. Find the coefficient of x^7 in the expansion of $\left(\dfrac{x^2}{2} - \dfrac{2}{x}\right)^8$.

6. A triangle has sides a, b and c and $a > b$. If the inequalities below are true, determine whether the triangle is acute, obtuse or right.

$$\begin{cases} \log_{a+b} C + \log_{a-b} C > 2\log_{a+b} C \cdot \log_{a-b} C \\ a - b > 1 \end{cases}$$

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[1] from Wolfram Research's "MathWorld" - mathworld.wolfram.com

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Section II – 40 minutes

1. Srinvasa Ramanujan was an incredible 20th century number theorist. Once when he was ill, G.H. Hardy visited him in the hospital and a nice story ensued:

"Once, in the taxi from London, Hardy noticed its number, 1729. He must have thought about it a little because he entered the room where Ramanujan lay in bed and, with scarcely a hello, blurted out his disappointment with it. It was, he declared, 'rather a dull number,' adding that he hoped that wasn't a bad omen. 'No, Hardy,' said Ramanujan, `it is a very interesting number. It is the smallest number expressible as the sum of two [positive] cubes in two different ways."[2]

Find one of the two representations of 1729 as the sum of two cubes.

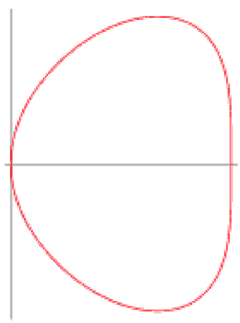
2. In a pizza shop you can buy a pizza with one or more of the following toppings: pepperoni, mushrooms, anchovies, pineapple, green peppers or onions. You can also order a pizza with no topping. Find the total number of different pizzas that you can order.

3. Simplify the following expression as much as possible: $2+\sqrt{2}+\dfrac{1}{2+\sqrt{2}}+\dfrac{1}{\sqrt{2}-2}$

4. For a given value of k the product of the roots of $x^2-3kx+2k^2-1=0$ is 7. Determine the nature of the roots (integral and/or rational, irrational or imaginary).

5. The diagonal of square I has length $a+b$. Find an expression for the perimeter of square II with twice the area of I.

6. There is a curve known as a "bean curve" due to its shape (see below). The equation of the curve is quartic: $x^4+x^2y^2+y^4=x\left(x^2+y^2\right)$. Estimate, to the nearest integer, the total area enclosed by this curve.



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[2] from Wolfram Research's "MathWorld" - mathworld.wolfram.com

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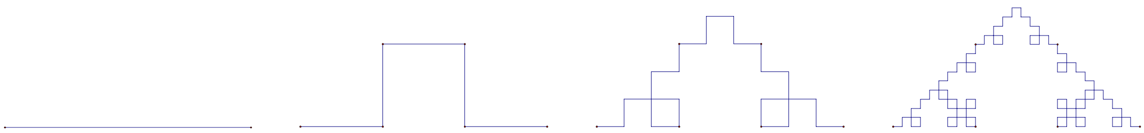
Section III – 40 minutes

1. A polygon P is such that each interior angle is $7\frac{1}{2}$ times the exterior angle at the same vertex. Find the number of sides of P.

2. Solve for x: $\left|\frac{5-x}{3}\right| < 2$, where $|a|$ is the absolute value of a.

3. A *Hat curve* is a fractal that is constructed as follows:



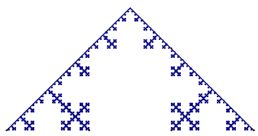
1st iteration
1 segment
length 1

2nd iteration
5 segments
length 5/3

3rd iteration

4th iteration

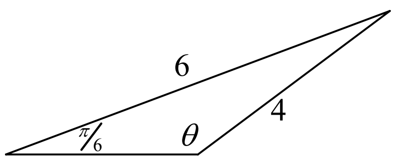
Find the length of the 8th iteration:



4. Solve for x: $\sqrt{5-x} = x\sqrt{5-x}$

5. Two women set out at the same time to walk towards each other from M and N, 72 km apart. The first woman walks at a rate of 4 km/hr. The second woman walks 2 km/hr the first hour, $2\frac{1}{2}$ km/hr the second hour, 3 km/hr the third hour, and so on in an arithmetic sequence. Determine how many hours it will take for the women to meet.

6. Given triangle ABC below, determine the value of sin θ.



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ANSWERS

Section 1

1. **56.25%**
2. **84 years**
3. **60 edges**
4. **2i**
5. **–14**
6. **obtuse (($a^2 > b^2 + c^2$)**

Section 2

1. **$1^3 + 12^3$ OR $9^3 + 10^3$**
2. **64 PIZZAS**
3. **2**
4. **IRRATIONAL ($\Delta = 8$)**
5. **$4a + 4b = 4(a + b)$**
6. **1**

Section 3

1. **17 SIDES**
2. **$-1 < x < 11$**
3. **78125/2187 OR $5^7/3^7$**
4. **$x = \{1, 5\}$**
5. **9 HOURS**
6. **3/4**