



# ECIS Mathematics League Fourier League 2006

**Section One**

**Calculators are not  
allowed**

**Time: 40 minutes**

1. Wojtek wants to mail a package which requires €1.53 in postage. He only has stamps of €0.05 and €0.08 denominations. Find the smallest number of stamps he can use to total €1.53.
2. The sum of the third and fourth terms in a sequence of consecutive integers is 47. Find the sum of the first five terms of the sequence.
3. Find all integer solutions to the system of equations below:

$$\begin{cases} a + bc = 1 \\ b + ac = 1 \\ c + ab = 1 \end{cases}$$

4. Which number below is the greatest?

$6^{100}$

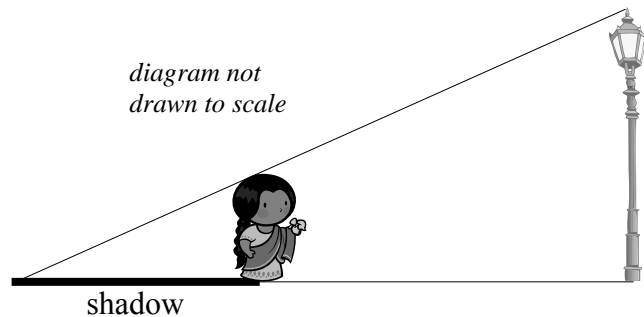
$5^{200}$

$4^{300}$

$3^{400}$

$2^{500}$

5. Anjali is standing 5 metres away from a street light. She is two metres tall and the light is six metres high. Find the length of Anjali's shadow.



6. A bus travels up a hill at an average speed of 50 km/hr. At what average speed would it have to travel down the hill to average 60 km/hr for the entire trip?



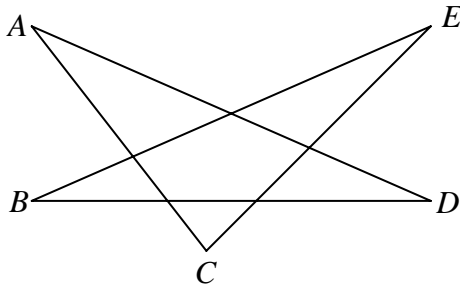
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**Section Two**

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**Time: 40 minutes**

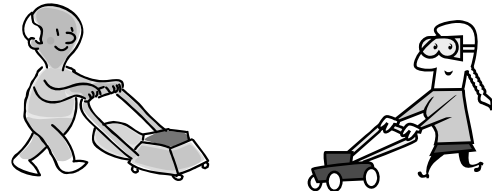
1. Find the sum of the measures of the acute angles  $A$ ,  $B$ ,  $C$ ,  $D$  and  $E$  in the figure below:



2. A cube measuring 100 units on each edge is painted only on the outside and cut into unit cubes. Find the number of cubes with paint only on two sides.
3. Find the largest constant  $C$  such that  $\sin x \geq Cx$  for  $0 \leq x \leq \frac{\pi}{2}$ ,  $x \in \mathbb{R}$ .

You can give your answer in terms of  $\pi$ .

4. It takes Michelle 60 minutes to mow her yard, but her father can only do it in 90 minutes. How long would it take them to mow the yard if they worked together using the two mowers?



5. A lattice point in the plane is a point both of whose coordinates are integers. How many lattice points (including the endpoints) are there on the line segment joining the points  $(2, 0)$  and  $(16, 203)$ ?
6. Five counterfeit coins are mixed with nine authentic coins. If two coins are drawn at random, find the probability that one is authentic and one is counterfeit.



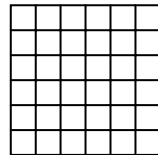
# ECIS Mathematics League Fourier League 2006

Section Three

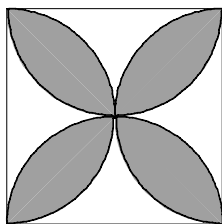
Calculators are not  
allowed

Time: 40 minutes

- Let  $A = 2006 + \frac{1}{2006}$ ,  $B = 2006 + \frac{1}{2006 + \frac{1}{2006}}$  and  $C = 2006 + \frac{1}{2006 + \frac{1}{2006 + \frac{1}{2006 + \frac{1}{2006}}}}$ . List  $A$ ,  $B$  and  $C$  in increasing order from least to greatest.
- Find the area of the largest triangle that can be inscribed in a semi-circle of radius  $r$ . You can leave your answer in terms of  $\pi$ .
- If  $\log_7(x^2) = \log_3(9)$ , find  $|x|$ .
- In the configuration below consisting of 36 one-by-one squares, how many total squares with horizontal and vertical sides can be formed using the points as vertices?



- The square below has sides of length  $2a$ . The shaded region consists of four bounded semi-circles which are in the interior of the square and have the four sides of the square as diameters. Find the shaded area in terms of  $a$  and  $\pi$ .



- A sum of 35 integers is  $S$ . Two digits in **one of the two digit numbers** are interchanged and a new sum  $T$  is produced. The difference  $S - T$  must be divisible by:

9

2

7

5

11



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Answers:

## Section 1

1. 21 stamps
2. 115
3.  $b = 0, a = 1, c = 1$   
 $b = 1, a = 1, c = 0$   
 $b = 1, a = 0, c = 1$
4.  $3^{400}$
5.  $x = 2.5\text{m}$
6. 75 km/h

## Section 2

1.  $180^\circ$
2. 1176 cubes with paint on two sides
3.  $C = \frac{2}{\pi}$
4. 36 min
5. 8 lattice points
6. 45/91

## Section 3

1.  $B < C < A$
2.  $A = r^2$
3. 7
4. 91
5.  $2a^2(\pi - 2)$ , *accept*  $\left(\frac{\pi a^2}{4} - \frac{a^2}{2}\right)$
6. 9