



ECIS Mathematics League Fourier League 2007

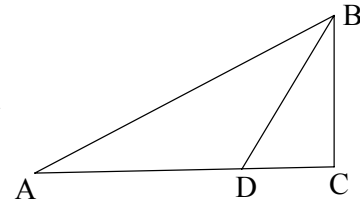
Section One

Calculators are not
allowed

Time: 40 minutes

Section I – 40 minutes

1. In the adjacent figure, triangle ABC has a right angle at C and $m\angle A = 20^\circ$. If \overline{BD} is the bisector of $\angle ABC$, then find the value of $m\angle BDC$.



2. If the point $(x, -4)$ lies on the straight line joining the points $(0, 8)$ and $(-4, 0)$ in the XY -plane, then find the value of x .
3. A positive number N is a *palindrome* if the integer obtained by reversing the sequence of digits of N is equal to N . The year **2772** is an example of a year with the following two properties:
- (a) It is a palindrome (when reversed it still reads 2772).
 - (b) It factors as a product of a 2-digit prime palindrome and a 3-digit composite palindrome. ($2772 = 11 \times 252$).

How many years in the millennium between **2000** and **3000** (including the year **2772**) satisfy these properties?

4. There is one integer value **for y** that is a solution **for both of the following equations:**

$$x^2 + y^2 - 16 = 0 \quad \text{and} \quad x^2 - 3y + 12 = 0$$

Find this value for y .

5. A restaurant offers three desserts, and exactly twice as many appetizers as main courses. A dinner consists of an appetizer, a main course and a dessert. What is the least number of main courses that the restaurant should offer so that a customer could have a different dinner each night in the year 2007?



6. Find the remainder when the number 2^{1000} is divided by 13.



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Section Two

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7. The lengths of the three sides of each of four triangles I, II, III and IV are as follows:

I 3, 4 and 5

III 7, 24 and 25

II 4, 7.5 and 8.5

IV 3.5, 4.5 and 5.5

State which triangles are right-angled (contain an angle 90°)?

8. The graph, G , of $y = \log_{10} x$ is rotated 90° counter-clockwise (anticlockwise) about the origin to obtain a new graph G' . What is an equation for G' ?

9. A merchant finds out that she can buy her goods for **8% less** while keeping her selling price fixed. If this happens, her profit, based on the buying price, would be increased to $(x+10)\%$ from her present profit of $x\%$.

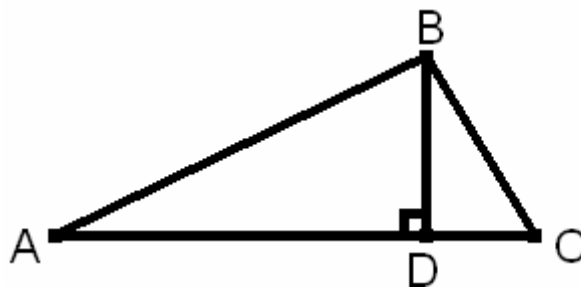
Find x .



10. Let $\Psi(x)$ represent the sum of the digits of the positive integer x . For example, $\Psi(8) = 8$ and $\Psi(123) = 1 + 2 + 3 = 6$. For how many two-digit values of x is $\Psi(\Psi(x)) = 3$?

11. From among $2^{\frac{1}{2}}$, $3^{\frac{1}{3}}$, $8^{\frac{1}{8}}$ and $9^{\frac{1}{9}}$, state the largest value.

12. Triangle ABC is a scalene triangle. BD is one of the altitudes (heights) of the triangle and it has **length 4**. Another altitude of the triangle ABC has **length 12**. If the length of the third altitude is also an integer, what is the biggest number it can be?





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Section Three

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Time: 40 minutes

13. **One hundred** International school students participated in the 2006 ECIS Math League, (aged 16 and 18) and their **mean score was 10**. The number of **16-year-olds** taking the exam was **50% more than the number of 18-year-olds**, and the **mean** of the **18-year-olds** was **50% higher** than that of the **16-year-olds**. What was the **mean score** of the 18-year-olds?
14. Using a table of a certain height, two identical blocks of wood are placed as shown in Figure 1. Length r is found to be **64 cm**. After rearranging the blocks as in Figure 2, length s is found to be **56 cm**. How high is the table?

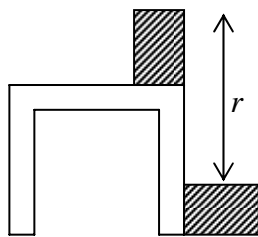


Figure 1

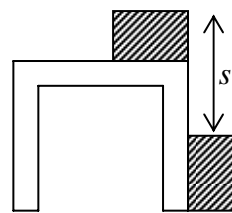


Figure 2

15. Find the sum of all real x such that $(2^x - 4)^3 + (4^x - 2)^3 = (4^x + 2^x - 6)^3$.
16. A circular disc with diameter D is placed on an 8×8 checkerboard (*chessboard*) with width D so that the centers coincide. Find the number of checkerboard (*chessboard*) squares that are **completely** covered by the disc.
17. The triple jump consists of an athlete leaping with one foot, then taking a step, and then jumping to the end. If a leaps equals b steps, c jumps equals d leaps, and e jumps equals f metres, then one metre equals how many steps?



18. A triangle has angles of 30° and 45° . The **side opposite the 45° angle** has length **8**. Find the length of the **side opposite the 30° angle**. You can leave your answer in terms of $\sqrt{2}$.



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Answers

1. 55°

2. 6

3. 8

4. 4

5. 8

6. 3

7. I, II and III

8. $y = 10^{-x}$

9. 15

10. 10

11. $3^{\frac{1}{3}}$

12. 5

13. 12.5

14. 60cm

15. $\frac{7}{2}$

16. 32

17. $\frac{bde}{acf}$

18. $4\sqrt{2}$